Research Note: Overselling with Opportunistic Cancellations

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Abstract
In many business sectors such as airlines, hotels, trucking, and media advertising, customers' arrivals and willingness to pay are uncertain. Managers must decide whether to quote a price low enough to guarantee early sales, or to quote a higher price and risk that some units remain unsold. In allocating capacity, they face a trade-off between two types of potential losses; (1) Yield loss—selling at a low price, and losing a better price later, and (2) Spoilage loss—waiting in vain to sell at a high price, and losing the opportunity of an earlier low price offer. Yield loss means that consumers who value the product most do not get to use it, and spoilage loss means that valuable products are wasted because no consumers get to use them. Sellers typically hedge against the risk of spoilage loss by selling some units early at low prices, and against the risk of yield loss by blocking some units in hope of selling them later at a high price.

In this paper we show that the use of overselling with opportunistic cancellations can increase expected profits and improve allocation efficiency. Under this strategy, the seller deliberately oversells capacity if high-paying consumers show up, even when capacity is already fully booked. The seller then cancels the sale to some low-paying customers while providing them with appropriate compensation.

We derive a new rule to optimally allocate capacity to consumers when overselling is used, and show that overselling helps limit the potential yield and spoilage losses. Yield loss is reduced because the seller can capture more high-paying customers by compensating low-paying customers who give up their right to the product. Spoilage loss is reduced because the compensation decreases the price spread perceived by the seller, and as a result, the seller is less anxious to speculate and “block” units. Overselling with opportunistic cancellations assures that the product will be sold to consumers who value it most. This means that “everybody wins”, and resources are allocated more efficiently than in conventional selling.

(Overselling; Overbooking; Yield Management; Yield and Spoilage Losses; Capacity Management)
1. Introduction
It has been more than 25 years since a gate agent refused to allow Ralph Nader to board a flight (a practice known as bumping) between Washington and Connecticut. Nader’s subsequent 1972 lawsuit served as a catalyst for a ruling that airlines must seek volunteers to give up their seats before bumping anyone. Typically carriers offer passengers anything from small vouchers to round-trip domestic tickets—even cash to give up a seat. According to *USA Today* (1998), in 1997 the 10 major U.S. airlines denied boarding to 53,546 passengers. But this number is small compared to the number of passengers who voluntarily gave up their seat for appropriate compensation. During the same year, these airlines “bought out” more than a million volunteers.

When capacity cannot accommodate the number of customers with reservations who show up, sellers face a situation of overselling (Smith et al. 1992). Such situations arise because of overbooking, the practice of accepting more reservations than the available capacity.1 Overbooking is used because customers with reservations often fail to claim what they had reserved (Belobaba 1989; Weatherford and Bodily 1992). It is estimated that in the airline industry, for example, 10%-15% of passengers do not claim the seat they reserved (Rothstein 1985, *USA Today* 1998). Based on historical records, airlines accept more reservations than their available capacity (Desiraju and Shugan 1999, Smith et al. 1992). In contrast, we show that overselling can be profitable, even if all consumers show up. Hence, sellers should use overselling even if they accurately predict no-show behavior.

More generally, we examine the circumstances under which it is profitable to oversell deliberately and to compensate customers who voluntarily give up their right to the product. We show that planned overselling is profitable when differences in consumers’ willingness to pay are high, when high-paying customers enter the market late, and when the number of customers is uncertain. These market characteristics are typical of a variety of industries, such as airlines, hotels, and media advertising (Littlewood 1972).

In such cases, the capacity available at a given time cannot be adjusted easily. Therefore booking the optimal mix of customers to maximize profits is tricky. Sellers face two types of potential losses in such situations: (1) *Yield loss*—selling at a low price and losing a better price later, and (2) *Spoilage loss*—waiting in vain to sell at a high price and losing the opportunity of an earlier low price offer. The challenge is when to lock in certain customers early, and when to refuse them despite the risk of capacity spoilage, in hope that bigger spenders will show up later.

To illustrate the dilemma, consider a travel agent inquiring about reserving rooms at the Mercure Hotel, demanding a low rate, and offering to pay in advance. There is a chance that a nearby hotel, which is hosting a marketing conference at the same time, will sell out. If this occurs, marketing scientists arriving without a secured room may well agree to pay top dollar for rooms at the Mercure. The hotel manager would like to secure the highest profit, but does not know whether that goal will be achieved by locking in guaranteed low-rate occupants, or holding out in hope of realizing higher profit from the marketing convention.

We present a strategy referred to as *overselling with opportunistic cancellations*, and examine circumstances under which it can reduce spoilage and yield losses. Under this strategy, sellers deliberately sell more units than available, and compensate low-paying customers who agree to give up their unit. To illustrate, let’s return to the Mercure Hotel anecdote: The Mercure manager offers the travel agent the room for $75 per night with the stipulation that her customers will agree to stay at another hotel if asked to do so, as long as they receive $75 in compensation (assuming the room at the new hotel costs $75). Assume that the agent and her customers agree. As it happens, an absent-minded marketing scientist appears at the Mercure hotel at the last moment and requests a room. When the desk clerk quotes a price of $200 per night, he immediately accepts. The manager refers one of the agent’s customers to a different hotel, pays her $75, and pockets an added profit of $50.

The model presented below leads to two new insights on the problem of allocating capacity to consumers. First, we show that a strategy of overselling with
opportunistic cancellations can be profitable when differences in consumers’ willingness to pay are high, when high-paying customers enter the market late, and when the number of customers is uncertain. Second, we derive an optimal decision rule for allocating capacity when overselling is used. The rule improves resource allocation efficiency by reducing the potential yield and spoilage losses. Reducing the potential yield loss means that consumers who value the product most are more likely to obtain it, and reducing the spoilage loss means wasting fewer valuable products.

2. The Model
Consider a seller with an available capacity of N units that must be sold with a deadline that applies both to the seller and to consumers. We assume that the marginal cost of each unit is zero, and that each unit can be sold in either Period 1 (now) or Period 2 (later). After Period 2 the product becomes worthless.

There are two types of consumers: early consumers with a low reservation price v, and late consumers with a higher reservation price V. The seller can sell units in Period 1 to early consumers at a low price, p. Alternatively, the seller can wait for late consumers who may show up in Period 2. We assume that the number of early consumers exceeds capacity, and refer to any unit that is not made available to these customers in the first period as blocked. The seller is uncertain how many late consumers will show up. The number of late consumers, L, who appear in the second period is a random variable with a distribution function denoted by F(L). For any late consumer that shows up, the seller can charge a higher price, P, for the product (P > p).

We assume that P is smaller than the late consumer’s reservation price (P < V).

The seller faces a dilemma: If the product is sold to an early consumer, and a late consumer appears, the seller incurs a “yield loss” equal to the price spread P − p. On the other hand, if the seller waits and the late consumer does not appear, the seller incurs a “spoilage loss,” equal to the price p. Therefore in each case the seller “leaves money on the table.” To show that overselling can help with this dilemma, we first analyze the case without overselling (conventional selling), and then analyze the case where capacity can be oversold (overselling).

2.1. Conventional Selling
Under conventional selling, overselling is not allowed. The seller has two alternatives: (a) sell units now to early consumers at the low price p, or (b) block units for late consumers.

Let b denote the number of units to be blocked for late consumers, 0 ≤ b ≤ N. The remaining units, N − b, are available in the first period, for the early consumers. Let q denote the probability that more than b − 1 late consumers will appear. Therefore the probability that the seller will be able to sell the bth unit at the high price P is

\[ q = 1 - F(b - 1). \] (1)

The following blocking rule holds.\(^3\)

**Blocking Rule Under Conventional Selling.** Under conventional selling, it is profitable to block the bth unit as long as

\[ q ≥ p/P. \] (2)

Intuitively, the seller should block a unit if the price spread P − p is large enough relative to the probability q of selling the unit at P.

2.2. Overselling with Opportunistic Cancellations
In this case units can be oversold. The seller offers early customers two alternatives:

(a) Buy the product at a price p, without a cancellation option by the seller.

(b) Buy the product at a price p, with a cancellation option giving the seller the right to cancel the sale for compensation R.\(^4\)

\(^3\)This is the basic result of Littlewood’s seat allocation problem (1972). Assume that b − 1 units are already blocked. The expected profit from blocking the bth unit is qP. If the unit is not blocked, the seller receives a price p. The expected profit from blocking the bth unit is larger than the profit from not blocking it, as long as (2) holds.

\(^4\)Reasons for offering these alternatives are: (1) early consumers can buy from other sellers who sell without a cancellation option, (2) rational consumers will not return to a seller offering insufficient compensation.
An early consumer can self-select the offer with the highest expected surplus. To induce self-selection of the cancellation option, the compensation should be large enough so that the expected surplus to early consumers from this option, \((1 - q)(v - p) + qR\), is at least as large as the expected surplus from conventional selling, \(v - p\), i.e., \((1 - q)(v - p) + qR \geq v - p\), or equivalently

\[ R \geq v - p, \quad (3) \]

So the seller will set \(R\) at \(v - p\), and exercise the cancellation option as long as there is a late consumer willing to pay a price \(P > v\).

The decision to use opportunistic cancellations impacts the decision of how many units to block. We show that the following blocking rule is optimal under an overselling strategy.

**Blocking Rule Under Overselling.** Under overselling, it is profitable to block the \(b\)th unit as long as

\[ q \geq p/v, \quad (4) \]

**Proof.** The expected profit from blocking the \(b\)th unit is \(qp\), which is the same as the expected profit from blocking this unit under conventional selling. The expected profit when this unit is not blocked under overselling is \(\Pi_{os} = (1 - q)p + q(P - R)\).\(^6\) The optimal expected profit is achieved when the seller maximizes this expression subject to constraint (3). Therefore, the seller will choose to set the compensation as low as possible, i.e., \(R^* = v - p\). Substituting \(R^*\) into \(\Pi_{os}\) we find that \(\Pi_{os}^* = p + q(P - v)\). Comparing \(\Pi_{os}^*\) to \(qp\), the expected profit from blocking the \(b\)th unit is larger than the profit from not blocking it, as long as (4) holds.

### 2.3. Comparing Overselling to Conventional Selling

We show in the Appendix that the following propositions hold when \(P > v\).

\(^5\)If \(P \leq v\) there is no arbitrage opportunity because the compensation \((v - p)\) is higher than the price spread \((P - p)\).

\(^6\)The first term is the profit if fewer than \(b\) consumers appear, so the seller obtains \(p\). The second term is the profit if more than \(b\) late consumers appear, so the seller obtains \(P\), but pays compensation \(R\) to the early consumer.

**Proposition 1.** The optimal number of blocked units is smaller under overselling than under conventional selling.

**Proposition 2.** Expected profit is higher under overselling than under conventional selling.

**Proposition 3.** Overselling is more economically efficient than conventional selling.

Intuitively, Proposition 1 holds because under overselling the potential yield loss is smaller than under conventional selling (the yield loss under overselling is the price spread minus compensation). Therefore, the seller prefers to reduce the hedge against potential yield loss by blocking a smaller number of units.

To understand why overselling leads to higher expected profit than conventional selling (Proposition 2) consider all units that will be blocked under conventional selling and unblocked under overselling. Clearly, for each of these units the expected compensation is lower than the expected spoilage and yield losses. Therefore, the expected profit from each of these units is higher under overselling, and that, in turn, leads to overall higher expected profit (see formal proof in the Appendix).

Consumers also benefit from overselling. Those with the highest value obtain the product as sales to low-paying consumers are canceled, and the low-paying consumers receive sufficient compensation. As a result “everybody wins,” and economic efficiency increases (Proposition 3).

### 3. Discussion

Our analysis emphasizes the opportunities embedded in the concept of overselling with opportunistic cancellations. We found some anecdotal evidence of its use in some industries, but these instances appear to be sporadic rather than systematic.\(^7\) This suggests that

\(^7\)Conversations with managers in several service sectors (especially airlines and hotels) provided us with some examples that can be considered as overselling with opportunistic cancellations. It appears that some companies have devised rules of thumb that approximate the systematic use of overselling in some cases. For example, airlines try to accommodate full-fare customers who show up just before the flight and offer compensations to customers who volunteer to give up their seats.
there is a missed opportunity to use overselling to increase profits. Some managers indicated that they would hesitate to apply this concept because of concerns about legal and public image aspects. However, as we have shown, overselling benefits customers (as well as sellers)—a fact that should be communicated to customers.

We have shown that under overselling, a seller blocks fewer units because the potential for yield loss is reduced. In a competitive environment, selling more in the first period implies less available capacity in the second period, which may help all the competitors raise prices in the second period. This raises another issue concerning price setting and our assumption that prices are exogenous: what would change if prices were endogenously determined? Overselling is likely to be more profitable and economically efficient compared to conventional selling as long as the compensation demanded by low-paying consumers is below the (optimal) price spread. In fact, in this case profits from overselling are likely to be greater than those we report.

We assumed that the sequence of customers’ arrival is strict, with low-paying consumers arriving early and high-paying consumers arriving late. This pattern is consistent with the demand conditions in many service industries (Kimes and Chase 1998). Yet, as long as the probability that high-paying consumers appear after low-paying consumers is high enough, overselling may well be profitable and economically efficient, even if the arrival sequence is not strict. In some industries such as fashion goods, however, high-paying customers appear first, and price skimming and clearance sales are likely to be profitable (Lazear 1986).

The model can be extended in additional directions. One can specify a distribution of reservation prices for early and late consumers. The incentive to oversell with opportunistic cancellation will still exist as long as the distribution of late consumers puts sufficient weight on the values representing higher willingness to pay than that of early consumers. Other extensions may include using continuous time, incorporating uncertainty about the willingness to pay of early consumers and the compensations required for them to accept opportunistic cancellations.

4. Concluding Remarks
To a naïve observer, the application of overselling could appear no different than the traditional application of overbooking. Overbooking is typically perceived as a rational reaction to the possibility of no-shows (i.e., cancellations by consumers), yet we show that overbooking may also occur because of the opportunity to reduce spoilage and yield losses. In fact, overbooking may occur even if all consumers show up, as we deliberately assumed in our model.

Marketers can mitigate the risks of yield and spoilage losses with a strategy of overselling with opportunistic cancellations. Under this strategy, the seller accepts reservations for more units than the available capacity and compensates low-paying consumers who agree to cancel their order. We derived a rule for allocating scarce resources when overselling is used, and showed that this rule is more effective in limiting spoilage and yield losses than the equivalent allocation rule when overselling is not used.

The logic of overselling can be extended beyond services such as lodging or transportation, to selling products such as a house or a piano, for which buyers vary in their willingness to pay. For instance, sellers of a house who receive a reasonable offer, may agree to sell the house for that price with a cancellation option under which the sellers will compensate the buyers if a substantially higher offer comes along within some time period.

Appendix

Proof of Proposition 1. Considering conditions (2) and (4) and recognizing that $\frac{p}{P} < \frac{p}{v}$ (when $P > v$), the number of units blocked under overselling is smaller than under conventional selling.

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Proof of Proposition 2. Let $b^*$ and $b^{*o}$ denote the optimal number of units blocked under conventional selling and overselling, respectively, and $\Pi(b^*)$ and $\Pi_o(b^*)$ the corresponding profits.

We need to show that

$$\Pi_o(b^*) > \Pi(b^*).$$

(A1)

By definition of $b^{*o}$, $\Pi_o(b^{*o}) \geq \Pi_o(b^*)$. Therefore (A1) holds if we show that

$$\Pi_o(b^*) > \Pi(b^*).$$

(A2)

Consider any unblocked unit $i$. The expected profit from that unit under overselling is $F(i - 1)p + [1 - F(i - 1)](P - R^*)$, and under conventional selling is $p$. Substituting $R^* = v - p$ in the expected profit from overselling, we obtain that the expected profit from overselling is higher for any unblocked unit.

Consider now any blocked unit $j$. In this case, the expected profit from overselling is equal to the expected profit under conventional selling because in both cases the unit is treated the same, i.e. it is only sold if a late consumer shows up.

Since the expected profit for any unblocked unit $i$ is higher from overselling, and the expected profit for any blocked unit $j$ is the same for overselling and conventional selling, (A2) is satisfied, and Proposition 4 holds.

Proof of Proposition 3. It follows immediately from Proposition 2 and the fact that by construction no consumer is worse off. (Under overselling the compensation to early consumers is sufficient to make them indifferent. Late consumers pay the same price, but have a higher probability of getting the product.)

References


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