Experimental Realization of Quantum-Resonance Ratchets at Arbitrary Quasimomenta

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Quantum-resonance ratchets associated with the kicked particle are experimentally realized for *arbitrary quasimomentum* using a Bose-Einstein condensate (BEC) exposed to a pulsed standing light wave. The ratchet effect for general quasimomentum arises even though both the standing-wave potential and the initial state of the BEC have a point symmetry. The experimental results agree well with theoretical ones which take into account the finite quasimomentum width of the BEC. In particular, this width is shown to cause a *suppression* of the ratchet acceleration for exactly resonant quasimomentum, leading to a saturation of the directed current.

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Understanding quantum transport in classically chaotic systems is a problem of both fundamental and practical importance. A wide variety of interesting quantum-transport phenomena have been discovered in simple but representative quantized models of Hamiltonian dynamics [1]. These phenomena either exhibit fingerprints of classical chaotic transport in a semiclassical regime or are purely quantum in nature. A paradigmatic and realistic class of model systems which has been studied most extensively, both theoretically and experimentally, consists of the periodically kicked rotor and variants of it [1–14]. These systems feature some of the most well-known phenomena in the field of quantum chaos, such as dynamical localization [1], i.e., the quantum suppression of classical chaotic diffusion, and the diametrically opposite phenomenon of *quantum resonance* (QR) [2]. The latter is a purely quantum quadratic growth of the mean kinetic energy in time occurring for special values of an effective Planck constant. The experimental realization of the kicked rotor using atom-optics techniques [3] has led to breakthroughs in the study of quantum chaos. Such experiments actually realize the kicked particle, not the kicked rotor, since atoms move on lines and not on circles like a rotor. However, the two systems can be exactly related due to the conservation of the particle *quasimomentum* β [6,8] (see also below).

An important concept introduced recently in classical and quantum Hamiltonian transport is that of a "ratchet." This is a spatially periodic system in which, without a biased force, a directed current of particles can be established. Ratchet models were originally proposed as mechanisms for some kinds of biological motors and as nanoscale devices for several applications [15]. In these contexts, the directed current is due to a spatial/temporal asymmetry combined with noise and dissipation. Ratchets based on classical physics have been implemented recently [16]. In a classical Hamiltonian system, dissipation is absent and noise is replaced by deterministic chaos. Here, a directed current of particles in the chaotic sea may arise under asymmetry conditions for a mixed phase space [17]. The corresponding quantized system may exhibit a significant ratchet behavior even in a fully chaotic regime [10,11,18]. Such a behavior, which occurs in a variant of the kicked rotor and can be related to the underlying classical dynamics, was observed recently in experiments using ultracold atoms [11]. Under *exact* QR conditions, theory predicts a purely quantum ratchet *acceleration*, i.e., a linear increase of the directed current (the mean momentum) in time [12,13]. An experimental observation of this phenomenon on a short time interval was reported quite recently for the usual kicked rotor [14], with $\beta = 0$. It is known, however, that the unavoidable experimental uncertainty in β strongly affects QR [6–8], and it is therefore natural to ask how it will affect the QR-ratchet acceleration.

In this Letter, we present an experimental realization of quantum ratchets associated with QR of the kicked particle for *arbitrary* values of the quasimomentum β. The experiments are conducted by exposing a Bose-Einstein condensate (BEC) to a pulsed optical standing wave which creates a potential $V(x) = V_{\max}\cos(Gx - \gamma)$. Here, x is the spatial coordinate along the standing wave, $G = 4\pi/\lambda$ is the "grating vector," λ is the light wavelength, and γ is an "offset" phase. The BEC is prepared in an initial state $\psi_0(x)$ with $|\psi_0(x)| \propto |\cos(Gx/2)|$, different from the pure momentum state normally used in quantum-chaos experiments [3–5,7,9,11]. Our experimental results agree well with a very recent general theory of QR ratchets [13], after including in this theory the effect of the finite quasimomentum width $\Delta\beta$ of the BEC. This width is shown to cause a *suppression* of the QR-ratchet acceleration for exactly "resonant" β [13], leading to a saturation of the directed current. Unlike other kinds of quantum ratchets [10,11,17,18], the QR ratchet exhibits fully *symmetric* features; both $V(x)$ and $\psi_0(x)$ have a point symmetry around some center. The directed current for general β depends strongly on the relative displacement between the two symmetry centers, here given by γ. Figure 1 shows the resonant current after 5 standing-wave pulses versus γ. Note that the current direction can be easily controlled

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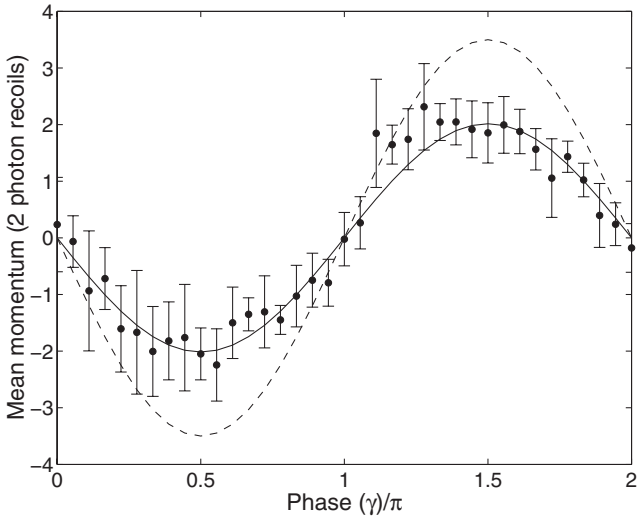

FIG. 1. Mean momentum vs phase angle γ for kicking strength $k = 1.4$, kick number $t = 5$, and resonant quasimomentum $\beta = 0.5$. The filled circles and associated error bars are from the experiment. The solid line corresponds to the theory (3) for a BEC with width $\Delta\beta = 0.056$. The dashed line corresponds to the ideal ($\Delta\beta = 0$) theory (1).

by γ and that the current vanishes for $\gamma = 0$ (coinciding symmetry centers).

We start by summarizing the basic theory (see more details in Refs. [6,8,13]). Using dimensionless quantities, the quantum kicked particle is described by the general Hamiltonian $\hat{H} = \hat{p}^2/2 + kV(\hat{x})\sum_t \delta(t' - t\tau)$, where $(\hat{p}, \hat{x})$ are momentum and position operators, k is the kicking strength, $V(x)$ is an arbitrary periodic potential, τ is the kicking period, and t' and t are the continuous and "integer" times. The units are chosen so that the particle mass is 1, $\hbar = 1$, and the period of $V(x)$ is 2π. The translational invariance of $\hat{H}$ in $\hat{x}$ implies the conservation of the quasimomentum β, $0 \leq \beta < 1$, in the time evolution of a Bloch wave packet $\exp(i\beta x)\psi(x)$. Here, $\psi(x)$ is a 2π-periodic function, so that one can consider x as an angle θ. Introducing the corresponding angular-momentum operator $\hat{N} = -id/d\theta$, the time evolution of $\psi(\theta)$ in one period τ is given by $\hat{U}_\beta\psi(\theta)$, where $\hat{U}_\beta = \exp[-ikV(\hat{\theta})]\exp[-i\tau(\hat{N} + \beta)^2/2]$. The latter operator describes a "β-kicked rotor" which is related to the kicked particle at fixed β by $\hat{p} = \hat{N} + \beta$; i.e., β is the "fractional" part of the particle momentum. QR of a β-kicked rotor is a quadratic growth of the mean kinetic energy at sufficiently large times t: $\langle\psi_t|\hat{N}^2/2|\psi_t\rangle \sim Dt^2$, where $\psi_t(\theta) = \hat{U}_\beta^t\psi_0(\theta)$ is an evolving wave packet and D is some coefficient. This behavior will generally occur provided two conditions are satisfied [8]: (a) The effective Planck constant $\tau/(2\pi)$ is a rational number l_0/q_0, where (l_0, q_0) are coprime integers; (b) β is also a rational number β_r characterized by two integers (r, g) defined as follows: g is the smallest integer such that $r = (\beta_r + gq_0/2)gl_0$ is integer. Then, the resonant quasimomentum $\beta = \beta_r$ can be written as $\beta = \beta_{r,g}$.

We shall focus on the case of integer $\tau/(2\pi) = l_0$ ($q_0 = 1$), corresponding to the "main" QRs. In this case, the time evolution of wave packets $\psi_t(\theta)$ under $\hat{U}_\beta$ can be exactly calculated for arbitrary potential $V(\theta) = \sum_m V_m \exp(-im\theta)$ and for any β [8]. One can then evaluate the current $\langle\hat{p}\rangle_t$, i.e., the mean momentum of the evolving Bloch wave. Using $\langle\hat{p}\rangle_t = \langle\hat{N}\rangle_t + \beta$, where $\langle\hat{N}\rangle_t \equiv \langle\psi_t|\hat{N}|\psi_t\rangle$, the *change* in the current relative to its initial value is given by $\Delta\langle\hat{p}\rangle_t = \langle\hat{N}\rangle_t - \langle\hat{N}\rangle_0$. This change is a measure of the ratchet effect induced by the kicking. Exact results for $\Delta\langle\hat{p}\rangle_t$ can be derived for general $\psi_0(\theta)$ [13], but we shall restrict ourselves here to the simplest initial wave packet giving ratchet effects: $\psi_0(\theta) = [1 + \exp(-i\theta)]/\sqrt{4\pi}$. The arbitrary potential $V(\theta)$ can be written, up to a real factor, as $V(\theta) = \cos(\theta - \gamma) + V_2(\theta)$, where $V_2(\theta)$ contains only harmonics of order $m \geq 2$. Then, by applying the general exact result (17) in Ref. [13] to our system (corresponding to $w = 0$ and $T = 1$ in [13]), we find for the initial wave packet above that

$$\Delta\langle\hat{p}\rangle_t = \frac{k}{2}\frac{\sin(\tau_\beta t/2)}{\sin(\tau_\beta/2)}\sin[(t+1)\tau_\beta/2 - \gamma], \qquad (1)$$

where $\tau_\beta = \pi l_0(2\beta + 1)$. Because of the simple choice of $\psi_0(\theta)$, this result is completely *independent* of $V_2(\theta)$. Thus, without loss of generality, henceforth we take $V(\theta) = \cos(\theta - \gamma)$. The denominator in (1) vanishes if $\tau_\beta = 2\pi r$ for some integer r. This corresponds precisely to a resonant value of $\beta = \beta_{r,g} = r/l_0 - 1/2 \bmod(1)$, with $g = 1$. For $\beta = \beta_{r,1}$, Eq. (1) reduces to a linear growth in time t, a ratchet acceleration: $\Delta\langle\hat{p}\rangle_{t,r} = -(k/2)\sin(\gamma)t$.

In an experimental realization of QR ratchets using a kicked BEC, the small but finite initial momentum width of the BEC can be important. Here, we consider a mixture of quasimomenta β', having a Gaussian distribution with average β and standard deviation $\Delta\beta$: $\Gamma_{\beta,\Delta\beta}(\beta') = (\Delta\beta\sqrt{2\pi})^{-1}\exp\{-(\beta' - \beta)^2/[2(\Delta\beta)^2]\}$. For small $\Delta\beta$, this is a good approximation of the actual initial momentum distribution [19]. The average of (1) over $\beta = \beta'$ with distribution $\Gamma_{\beta,\Delta\beta}(\beta')$ can be exactly calculated:

$$\langle\Delta\langle\hat{p}\rangle_t\rangle_{\Delta\beta} = \frac{k}{2}\sum_{s=1}^{t}\sin(\tau_\beta s - \gamma)\exp[-2(\pi l_0\Delta\beta s)^2]. \qquad (2)$$

Unlike (1), the expression (2) tends to a well-defined finite value as $t \to \infty$, for *all* β. In particular, for resonant $\beta = \beta_{r,1}$ with $\tau_\beta = 2\pi r$, Eq. (2) reduces to

$$\langle\Delta\langle\hat{p}\rangle_t\rangle_{r,\Delta\beta} = -\frac{k}{2}\sin(\gamma)\sum_{s=1}^{t}\exp[-2(\pi l_0\Delta\beta s)^2]. \qquad (3)$$

The result (3) implies a suppression of the ratchet acceleration above, which is recovered as $\Delta\beta \to 0$. In practice,

for sufficiently small $\Delta\beta$, the value of $\langle\Delta\langle\hat{p}\rangle_t\rangle_{\Delta\beta}$ for β close to $\beta_{r,1}$ is much larger than that for generic β, except when $|\sin(\gamma)|$ is very small.

It should be noted that both the potential $V(\theta) = \cos(\theta - \gamma)$ and the initial wave packet $\psi_0(\theta) = [1 + \exp(-i\theta)]/\sqrt{4\pi}$ have a point symmetry: $V(2\gamma - \theta) = V(\theta)$ (inversion around $\theta = \gamma$) and $\psi_0^*(-\theta) = \psi_0(\theta)$ (inversion around $\theta = 0$ accompanied by time reversal). The results (1)–(3) depend on the relative displacement γ between the symmetry centers of $V(\theta)$ and $\psi_0(\theta)$. For $\gamma = 0$, the system is "symmetric" and there is no resonant ratchet effect, $\langle\Delta\langle\hat{p}\rangle_t\rangle_{r,\Delta\beta} = 0$. At fixed k, $|\langle\Delta\langle\hat{p}\rangle_t\rangle_{r,\Delta\beta}|$ is largest for $\gamma = \pm\pi/2$, values of γ which may be viewed as corresponding to "maximal asymmetry" situations. The direction of the change (3) in the current is given by the sign of $-\sin(\gamma)$. We shall henceforth use $l_0 = 1$, corresponding to the "half-Talbot time" so that the only resonant value of β is $\beta = 0.5$. It is easy to see that $\langle\hat{p}\rangle_0 = \langle\hat{N}\rangle_0 + \beta = 0$ for $\beta = 0.5$ and then $\Delta\langle\hat{p}\rangle_t = \langle\hat{p}\rangle_t$.

Our experiments were carried out using the all-optical BEC apparatus described in Ref. [5]. After creating a BEC of ~50 000 ^{87}Rb atoms in a focused CO_2 laser beam, we applied a series of optical standing-wave pulses from a diode laser beam (6.8 GHz red detuned from the 780 nm laser cooling transition) propagating at 52° to the vertical. Through an ac-Stark shift, the standing wave changed the energy of the atoms by an amount proportional to the light intensity. The resulting spatially periodic phase modulation of the BEC wave function acted as a phase grating. Each of the two counterpropagating laser beams which comprised the standing wave passed through an acousto-optic modulator (AOM) driven by an arbitrary waveform generator. This enabled us to control the frequency and phase of each of the beams. Adding two counterpropagating waves differing in frequency by Δf results essentially in a standing wave that moves with a velocity $v = 2\pi\Delta f/G$, where G is the grating vector. The initial momentum or quasimomentum β of the BEC relative to the standing wave is proportional to v. Thus, by varying Δf, we could set arbitrarily the value of β and also compensate for the effect of the gravitational acceleration along the standing wave (the experiments were done in a free-falling frame).

In order to prepare the initial state, the first standing-wave pulse was relatively long, having a duration of 38 μs. This pulse Bragg diffracted the atoms into a superposition of two plane waves [20]: $|\psi_0\rangle = [|P = 0\rangle + |P = \hbar G\rangle]/\sqrt{4\pi}$, where P is the nonscaled momentum. The second and subsequent pulses of the standing wave were short enough to be in the Raman-Nath regime. These pulses diffracted the atoms into a wide spread of momentum states and enabled the realization of a kicked-rotor system. The value of the kicking strength k was measured by subjecting the BEC to one kick and comparing the populations of various diffraction orders; we used $k \sim 1.4$. By varying the phase of the RF waveforms driving the AOMs, we were able to shift the position of the standing wave for the kicked rotor relative to the standing wave used in the Bragg-state preparation. This is the phase γ in Eqs. (1)–(3). Finally, in order to probe the momentum distribution, we waited 8 ms and then imaged the atoms in absorption. Measurements of the initial momentum width of the BEC using a time-of-flight technique gave an upper bound to $\Delta\beta$ of 0.1. The slow expansion of the BEC and the finite resolution of our imaging system made it difficult to measure this quantity more precisely.

We have performed a comprehensive experimental study of the mean momentum of the BEC as function of several variables. Error bars for all the data were accurately determined by repeated measurements of the mean momentum at fixed values of the parameters. The results are presented in Figs. 1–3 and are compared with the theory above. These figures show the dependence of the mean momentum on the phase γ for $t = 5$ and resonant $\beta = 0.5$ (Fig. 1), its dependence on t for $\gamma = \pi/2$ and $\beta = 0.5$ (Fig. 2), and the dependence of the mean-momentum change on β for $t = 5$ and $\gamma = \pm\pi/2$ (Fig. 3). The solid line in the figures corresponds to Eq. (2) or Eq. (3). The dashed line in Figs. 1 and 3 corresponds to the nonaveraged theory (1). We can see that the experimentally determined mean momentum in all the figures is well fitted by the theory (2) or (3) for the *same* value of the width $\Delta\beta$, $\Delta\beta = 0.056$. This value of $\Delta\beta$ is also consistent with what can be measured directly using time-of-flight. These facts indicate good agreement of the experimental results with the QR-ratchet theory above. Thus, the clear saturation of the mean momentum in Fig. 2 provides experimental evidence for the suppression of the resonant ratchet acceleration. The

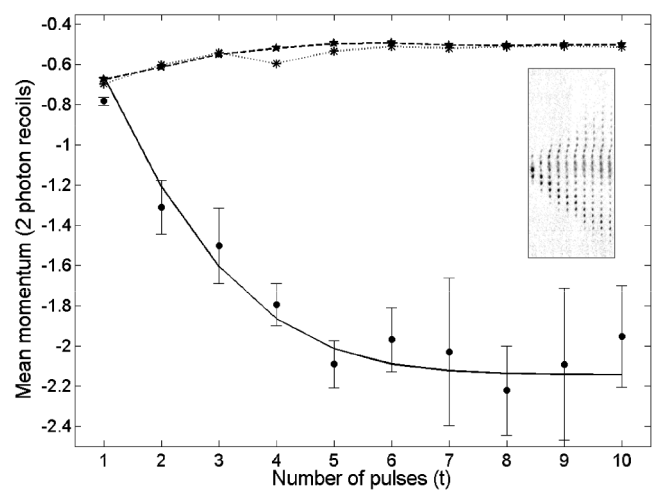

FIG. 2. Mean momentum vs kick number t for $k = 1.4$, $\gamma = \pi/2$, and $\beta = 0.5$. The filled circles and error bars are experimental data, and the solid line corresponds to Eq. (3) ($\Delta\beta = 0.056$). The dashed and dotted lines are the classical mean momentum $\langle p_t\rangle$ with average taken over two different initial ensembles in phase space (see text for more details). The inset shows the time-of-flight images of the BEC vs t.

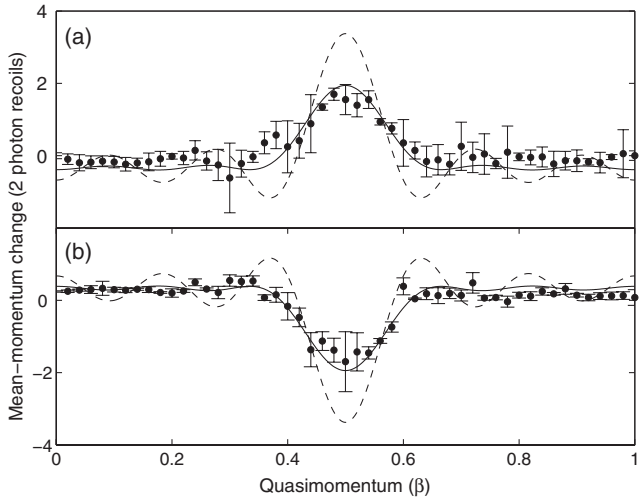

FIG. 3. Mean-momentum change after $t = 5$ kicks vs β for $k = 1.4$ and (a) $\gamma = -\pi/2$, (b) $\gamma = \pi/2$. The filled circles and error bars are experimental data; the dashed and solid lines correspond, respectively, to Eqs. (1) and (2) ($\Delta\beta = 0.056$).

inset in Fig. 2 plots the time-of-flight images of the kicked BEC as time increases. Note that the distribution of the momentum states is not symmetric and is weighted towards the negative diffraction orders, as expected from the mean-momentum values.

It is instructive to compare the behavior of the quantum mean momentum in Fig. 2 with that of its closest classical analogue. This is $\langle p_t \rangle$, where p_t is the classical momentum at time t, given by a standard map, and $\langle\rangle$ denotes average over an ensemble of initial conditions (p_0, x_0) whose distribution in phase space is the same as that featured by the quantum averages in (3). Thus, the distribution function of x_0 is $\phi(x_0) = \cos^2(x_0/2)/\pi$, equal to the probability density $|\psi_0(x)|^2$ for the initial wave packet; p_0 is distributed like $\langle \hat{p} \rangle_0 = \langle \hat{N} \rangle_0 + \beta'$, where $\langle \hat{N} \rangle_0 = -0.5$ and the distribution of β' around $\beta = 0.5$ is the Gaussian $\Gamma_{\beta,\Delta\beta}(\beta')$ above with $\Delta\beta = 0.056$. One has $\langle p_0 \rangle = \langle \langle \hat{p} \rangle_0 \rangle_{r,\Delta\beta} = 0$. The results for $\langle p_t \rangle$ are plotted in Fig. 2 (dashed line), showing a saturation to a value clearly *different* from the quantum one. In fact, this classical saturation is *not* due to averaging over p_0 or β', as in the quantum case, but to the relaxation of the initial nonuniform distribution $\phi(x_0)$ to an almost uniform one in phase space, which is fully chaotic for $k = 1.4$. The transient nonuniformity during the relaxation process causes $\langle p_t \rangle$ to acquire its nonzero saturated value. To show explicitly that the average over p_0 is classically not essential, we plot in Fig. 2 also $\langle p_t \rangle$ with the average taken over an ensemble with constant $p_0 = 0$ and with x_0 distributed as above (dotted line). We see that $\langle p_t \rangle$ saturates to the same value. All this demonstrates that the experimental results in Fig. 2, as well as those in Figs. 1 and 3, reflect *purely* quantum phenomena for a nonsmall value of the effective Planck constant, $\tau/(2\pi) = 1$.

In conclusion, we have experimentally realized QR ratchets for arbitrary quasimomentum β. These purely quantum ratchets are unique in their strong dependence on a *conserved quantum* entity (β, see Fig. 3) and in their fully *symmetric* features on which they also depend strongly through the phase γ (Fig. 1). The consideration of general β is necessary to account for the finite width $\Delta\beta$ of the BEC. We have shown that this width is the main reason for the suppression of the resonant ratchet acceleration, a distinctive feature of ideal QR ratchets [12,13]. A fingerprint of this acceleration for finite $\Delta\beta$ is the pronounced ratchet effect around resonant β (Fig. 3). To increase further the resonant directed current, $\Delta\beta$ has to be decreased below the value used in this work. We hope to achieve this in the future by improving our present experimental setup.

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