Quantum Accelerator Modes in the Absence of Gravity

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Abstract – *The experimental study of cold atoms kicked by a pulsed optical standing wave and falling under gravity has led to the important discovery of the "quantum accelerator modes" (QAMs). It is shown that QAMs in the absence of gravity can feature a rotational quasiregularity which cannot be found in the usual QAMs. Thus, QAMs with non-accelerating quasiregular segments may be used to control the quantum motion of atoms in a way which is not possible in the presence of gravity.*

***Keywords**: Control, Experimental Quantum Chaos, Quantum Accelerator Modes, Quantum Resonances, Quasiclassics, Rotating Accelerator-Mode Islands.*

I. Introduction

The atom-optics experimental realization of the quantized kicked rotor [1], a paradigmatic model system of Hamiltonian dynamics, opened new horizons in the field of Quantum Chaos. Experiments in which cold atoms were kicked when falling under gravity have led to the important discovery of the purely quantum "accelerator modes" (QAMs) in the free-falling frame [2]. QAMs occur for values of a scaled Planck's constant τ close to quantum resonance. These values of τ were shown [3] to correspond to a "quasiclassical" regime, in which the exact quantum dynamics can be approximately described by the classical map

$$p_{t+1} = p_t + K\sin(x_t) + 2\pi\Omega, \quad x_{t+1} = x_t + p_{t+1} \bmod(2\pi), \tag{1}$$

where (p, x) are the atom momentum and position, t is an "integer" time, K is the kicking strength, and Ω is proportional to the gravity force. Then, wave packets initially *trapped* in accelerator-mode islands (AIs) of (1) lead to the QAMs. This theoretical explanation [3] of QAMs was verified by various experiments and a multitude of new high-order QAMs were observed [4].

II. QAMs in the Absence of Gravity

For $\Omega = 0$, Eqs. (1) reduce to the standard map, describing the quasiclassical regime in the absence of gravity [5]. As far as we are aware, QAMs in this case have not been yet observed experimentally, apparently due to the much focus on the $\Omega \neq 0$ case until now. However, the $\Omega = 0$ QAMs are most interesting to study since they can be basically different in nature from the $\Omega \neq 0$ ones, due to the difference between the corresponding AIs. This difference can be seen most clearly by considering the case of integer $\Omega \neq 0$ (the arguments below can be easily extended to the case of general rational Ω, treated in [6]). In this case, (1) and the standard map obviously coincide if both maps are restricted to the basic torus $T^2 = [0, 2\pi)^2$ in the (p, x) phase space. Thus, for sufficiently small K, there exist islands in almost any rotational resonance [with, say, winding number ν] of (1) on T^2. Under the original (unrestricted) map (1), such an island corresponds to an AI: In *one* iteration of (1), an island in resonance ν will accelerate by jumping to resonance $\nu' = \nu + \Omega \neq \nu$.

For $\Omega = 0$, on the other hand, an interesting kind of AI has been discovered quite recently [7]. This is an AI which *completes* one or more rotations in a given resonance ν (where each rotation consists of more than one iteration) *before* jumping to another resonance ν'. Such *rotating* AIs (RAIs) feature a "horizontal" quasiregularity [7] which cannot be found in $\Omega \neq 0$ AIs. This quasiregularity of RAIs will have very clear quantum manifestations in the corresponding QAMs in a quasiclassical regime. QAMs on RAIs should be experimentally observable using atom-optics techniques, at least for sufficiently small $K > 0.9716$ and relatively large RAIs (see examples in [7]). By introducing non-accelerating quasiregular segments in a QAM, one can *control* the quantum motion of atoms in a way which is not possible for $\Omega \neq 0$.

III. Conclusion

We thus hope that this paper will motivate both the experimental and the theoretical study of QAMs on the RAIs in the standard map, i.e., the map (1) in the absence of gravity ($\Omega = 0$).

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