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QUANTUM-RESONANCE RATCHETS: THEORY AND EXPERIMENT

I. DANA and V. B. ROITBERG
Minerva Center and Department of Physics,
Bar-Ilan University, Ramat-Gan 52900, Israel

V. RAMAREDDY, I. TALUKDAR and G. S. SUMMY
Department of Physics, Oklahoma State University,
Stillwater, Oklahoma 74078-3072, USA

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A theory of quantum ratchets for a particle periodically kicked by a general periodic potential under quantum-resonance conditions is developed for arbitrary values of the conserved quasimomentum β. A special case of this theory is experimentally realized using a Bose–Einstein condensate (BEC) exposed to a pulsed standing light wave. While this case corresponds to *completely symmetric* potential and initial wave-packet, a *purely quantum* ratchet effect still arises from the generic *noncoincidence* of the symmetry centers of these two entities. The experimental results agree well with the theory after taking properly into account the finite quasimomentum width of the BEC. This width causes a *suppression* of the ratchet acceleration occurring for "resonant" β, so that the mean momentum saturates to a finite ratchet velocity, strongly pronounced relative to that for nonresonant β.

Keywords: Quantum Hamiltonian ratchets; kicked particle; quantum resonances; quasimomentum; quantum-resonance ratchets under symmetry conditions; atom optics; Bose–Einstein condensates.

1. Introduction

"Ratchets" are usually conceived as spatially periodic systems with noise and dissipation in which an unbiased (zero-mean) external force can lead to a directed current of particles; this is due to the breaking of some spatial and/or temporal symmetry [Reimann, 2002; Astumian & Hänggi, 2002]. Recently [Flach *et al.*, 2000; Schanz *et al.*, 2005], the ratchet concept was introduced in Hamiltonian dynamics, where dissipation is absent and noise is replaced by deterministic chaos. It was shown that necessary conditions for the classical chaotic region to exhibit a directed current are asymmetry and a *mixed* phase space, featuring "transporting" (e.g. accelerator-mode) stability islands. Quantized Hamiltonian ratchets may exhibit significant currents also under *full-chaos* conditions and have been studied in many works [Schanz *et al.*, 2005; Lundh & Wallin, 2005; Gong & Brumer, 2006; Jones *et al.*, 2007; Denisov *et al.*, 2007; Dana & Roitberg, 2007; Sadgrove *et al.*, 2007; Dana *et al.*, 2008; Dana, 2008]. Quantum-chaotic ratchets may be viewed as the deterministic counterpart of the quantum Brownian-motion ratchets [Reimann *et al.*, 1997; Goychuk & Hänggi, 2001]. Particularly interesting cases are strong quantum regimes corresponding to "quantum resonances" (QRs) [Lundh & Wallin, 2005; Dana & Roitberg, 2007; Sadgrove *et al.*, 2007; Dana *et al.*, 2008]. QR in "kicked" systems is a *purely quantum* asymptotic quadratic growth in time of the mean kinetic energy for rational values of a scaled Planck constant [Izrailev, 1990; Dana & Dorofeev, 2006]. Under QR conditions, an asymmetry may lead to a "ratchet acceleration", i.e.

a linear growth in time of the mean momentum in some preferred direction, despite the absence of a biased kicking force [Lundh & Wallin, 2005; Dana & Roitberg, 2007]. Quite recently [Sadgrove *et al.*, 2007; Dana *et al.*, 2008], "main" QR ratchets in a strong quantum regime have been experimentally realized using atom-optics techniques with Bose–Einstein condensates (BECs). A good agreement was found between theory and experiment, after properly adapting the theoretical results to the experimental conditions [Dana *et al.*, 2008]. A unique feature of these QR ratchets is that both the kicking potential and the initial wave packet are *completely symmetric*. It is the *noncoincidence* of their symmetry centers that causes this new kind of quantum ratchet effect.

This paper is an extended version of our Letter [Dana *et al.*, 2008], mainly concerning the theoretical aspects which are presented in detail in Secs. 2 and 3. The experimental realization and related issues are summarized in Sec. 4. Conclusions are given in Sec. 5.

2. Quantum Resonances of the Kicked Particle

The quantum kicked particle is described by the general Hamiltonian:

$$\hat{H} = \frac{\hat{p}^2}{2} + kV(\hat{x})\sum_t \delta(t' - t\tau), \tag{1}$$

where $(\hat{x}, \hat{p})$ are the position and momentum of the particle, k is a nonintegrability parameter, $V(\hat{x})$ is a periodic potential, t takes all the integer values, t' is the usual (continuous) time, and τ is the kicking period. The units are chosen so that the particle mass is 1, the Planck constant $\hbar = 1$, and the period of $V(\hat{x})$ is 2π. The one-period evolution operator for (1), from $t' = t + 0$ to $t' = t + \tau + 0$, is given by

$$\hat{U} = \exp[-ikV(\hat{x})]\exp\left(\frac{-i\tau\hat{p}^2}{2}\right). \tag{2}$$

The translational invariance of (2) in $\hat{x}$ implies the conservation of a quasimomentum β ($0 \leq \beta < 1$): The application of $\hat{U}$ on the special initial state of a Bloch function $\Psi_\beta(x) = \exp(i\beta x)\psi_\beta(x)$ [with 2π-periodic $\psi_\beta(x)$], results in a Bloch function $\Psi'_\beta(x) = \exp(i\beta x)\psi'_\beta(x)$ associated with the same value of β. Here $\psi'_\beta(x) = \hat{U}_\beta\psi_\beta(x)$, where

$$\hat{U}_\beta = \exp[-ikV(\hat{x})]\exp\left[\frac{-i\tau(\hat{p}+\beta)^2}{2}\right]. \tag{3}$$

The restriction of the operator (3) to 2π-periodic functions $\psi_\beta(x)$ allows one to interpret x as an angle θ and $\hat{p}$ as an angular-momentum operator $\hat{N} = -id/d\theta$ with integer eigenvalues n. Then, with the replacement $(\hat{x}, \hat{p}) \to (\theta, \hat{N})$, one can view (3) as the one-period evolution operator for a "β-kicked rotor" (β-KR).

The usual KR ($\beta = 0$) is known to exhibit the purely quantum phenomenon of quantum resonance (QR) for rational values of the effective Planck constant $\tau/(2\pi)$ [Izrailev, 1990]. This is an asymptotic quadratic growth of the kinetic-energy expectation value in time for arbitrary initial wave packet $\psi_0(\theta)$: $\langle\psi_t|\hat{N}^2/2|\psi_t\rangle \sim t^2$ for large t. The basic origin of QR is a band quasienergy spectrum of the one-period evolution operator, due to the invariance of this operator under translations $\hat{T}_q = \exp(-iq\theta)$ by q in the angular momentum $\hat{N}$ [Izrailev, 1990]; here q must be an integer since θ is an angle ($0 \leq \theta < 2\pi$). The translational invariance of β-KRs, $\beta \neq 0$, is expressed by $[\hat{U}_\beta, \hat{T}_q] = 0$, where $\hat{U}_\beta$ is given by Eq. (3) with $(\hat{x}, \hat{p}) \to (\theta, \hat{N})$. Using the fact that $\hat{N}$ has integer eigenvalues, one easily finds that $[\hat{U}_\beta, \hat{T}_q] = 0$ is satisfied only if

$$\frac{\tau}{2\pi} = \frac{l}{q}, \quad \beta = \frac{r}{l} - \frac{q}{2} \bmod(1), \tag{4}$$

where l and r are integers. Equation (4) gives the rationality condition for $\tau/(2\pi)$ and a formula for the general "resonant" values of β. For definiteness and without loss of generality, we assume that l and q are positive. Let us now write $l = gl_0$ and $q = gq_0$, where l_0 and q_0 are coprime positive integers and g is the greatest common factor of (l, q); the value of $\tau/(2\pi) = l_0/q_0$ will be kept fixed in what follows. It is then clear that β in Eq. (4) can take *any rational* value $\beta_{\rm r}$ in $[0, 1)$ since g can be always chosen so that $r = (\beta_{\rm r} + gq_0/2)gl_0$ is an integer. For given $\beta = \beta_{\rm r}$, we shall choose g as the *smallest* positive integer satisfying the latter requirement, so as to yield the minimal values of $l = gl_0$ and $q = gq_0$. See [Dana & Dorofeev, 2006] for more details.

3. Main Quantum-Resonance Ratchets

The Hamiltonian (1) describes a ratchet system since the force $f(x) = -kdV/dx$ is obviously unbiased, $\int_0^{2\pi} f(x)dx = 0$, due to the 2π-periodicity of $V(\hat{x})$. Intuitively, the QR behavior $\langle\psi_t|\hat{N}^2/2|\psi_t\rangle \sim t^2$ may imply the linear growth $\langle\psi_t|\hat{N}|\psi_t\rangle \sim t$ under some conditions. The latter growth is a *QR-ratchet acceleration*. To study this more precisely, we shall focus on the "main" QRs with $\tau = 2\pi l_0$ ($q_0 = 1$). For these QRs, the quantum evolution of wave packets under (3) can be exactly calculated for arbitrary values of β, i.e. not just for the resonant values given by Eq. (4). In fact, since $\hat{N}$ has integer eigenvalues, the relation $\exp(-i\pi l_0 \hat{N}^2) = \exp(-i\pi l_0 \hat{N})$ holds, so that (3) with $(\hat{x}, \hat{p}) \to (\theta, \hat{N})$ can be expressed for $\tau = 2\pi l_0$ as follows:

$$\hat{U}_\beta = \exp[-ikV(\theta)] \exp[-i\tau_\beta \hat{N}], \tag{5}$$

where $\tau_\beta = \pi l_0(2\beta + 1)$ and an irrelevant phase factor has been neglected. We note that the second exponential operator in (5) is just a shift in θ. Thus, the result of successive applications of (5) on an initial wave packet $\psi_0(\theta)$ can be written in a closed form:

$$\begin{aligned}\psi_t(\theta) &= \hat{U}_\beta^t \psi_0(\theta) \\ &= \exp[-ik\overline{V}_{\beta,t}(\theta)]\psi_0(\theta - \tau_\beta t),\end{aligned} \tag{6}$$

where

$$\overline{V}_{\beta,t}(\theta) = \sum_{s=0}^{t-1} V(\theta - \tau_\beta s). \tag{7}$$

Using the Fourier expansion

$$V(\theta) = \sum_m V_m \exp(-im\theta) \tag{8}$$

in Eq. (7), we find that

$$\overline{V}_{\beta,t}(\theta) = \sum_m V_m \frac{\sin\left(\frac{m\tau_\beta t}{2}\right)}{\sin\left(\frac{m\tau_\beta}{2}\right)} e^{im\tau_\beta(t-1)/2} \exp(-im\theta). \tag{9}$$

We now show that an asymptotic linear growth of $\langle \hat{N} \rangle_t \equiv \langle \psi_t | \hat{N} | \psi_t \rangle$ generically takes places precisely at the resonant values of β in (4):

$$\langle \hat{N} \rangle_t \approx \langle \hat{N} \rangle_0 + It, \tag{10}$$

where I is the momentum current (acceleration) for which the explicit general formula (14) below will be derived. We start from the general expansion

$$|\psi_0(\theta)|^2 = \frac{1}{2\pi} \sum_m C(m) \exp(im\theta), \tag{11}$$

where $C(m) = \sum_n \tilde{\psi}_0(m+n)\tilde{\psi}_0^*(n)$ are correlations of the initial wave packet in its angular-momentum representation $\tilde{\psi}_0(n)$. Using (6), (9) and (11), we get

$$\begin{aligned}\langle \hat{N} \rangle_t &= -i \int_0^{2\pi} \psi_t^*(\theta) \frac{d\psi_t(\theta)}{d\theta} d\theta \\ &= \langle \hat{N} \rangle_0 + ik \sum_m m V_m C(m) \\ &\quad \times \frac{\sin\left(\frac{m\tau_\beta t}{2}\right)}{\sin\left(\frac{m\tau_\beta}{2}\right)} e^{-im\tau_\beta(t+1)/2},\end{aligned} \tag{12}$$

where normalization of $\psi_0(\theta)$ is assumed, $\int_0^{2\pi} |\psi_0(\theta)|^2 d\theta = 1$. Now, a linear growth (10) in t can arise only if $m\tau_\beta/2 = r_m \pi$ for some $m \neq 0$, where r_m is integer; then, the contribution of the last two terms in (12) is just equal to t. Using $\tau_\beta = \pi l_0(2\beta + 1)$ in $m\tau_\beta/2 = r_m\pi$, we find that β must satisfy

$$\beta = \frac{r_m}{ml_0} - \frac{1}{2} \bmod(1). \tag{13}$$

By comparing (13) with Eq. (4), in which $l = gl_0$ and $q = gq_0 = g$ for some "minimal" g (see above), we see that (13) gives just a resonant value of β: m is some multiple of g ($m = jg$, j integer) and $r_m = j[r + l_0 g(1-g)/2]$ for some integer r. Then, by collecting all the terms with $m = jg$ in (12), we obtain a formula for the current I in (10):

$$I = -2kg \sum_{j>0} j \mathrm{Im}[V_{jg} C(jg)]. \tag{14}$$

Thus, for given resonant quasimomentum $\beta = \beta_\mathrm{r}$, $I \neq 0$ only if there exist sufficiently high harmonics V_m and correlations $C(m)$, with $m = jg$, and the sum of the corresponding terms in (14) is nonzero. These conditions are satisfied by generic $V(\theta)$ and $\psi_0(\theta)$. A very simple case of $I = 0$ is when $V_m C(m)$ is real for all m. This occurs, e.g. when the system is "symmetric", i.e. when both $V(\theta)$ and $\psi_0(\theta)$ have a point symmetry around the *same* center, say $\theta = 0$: $V(-\theta) = V(\theta)$ and $\psi_0(-\theta) = \pm\psi_0(\theta)$ (inversion) or $\psi_0^*(-\theta) = \pm\psi_0(\theta)$ (inversion accompanied by time reversal); this implies that V_m and $C(m)$ [see Eq. (11)] are both real.

As an example, we consider the simple case of $g = 1$ and initial wave packets $\psi_0(\theta) = \{1 + \exp[i(\gamma_0 - \theta)]\}/\sqrt{4\pi}$ for all phases γ_0. Clearly, $\psi_0^*(2\gamma_0 - \theta) = \psi_0(\theta)$, i.e. $\psi_0(\theta)$ has symmetry center at $\theta = \gamma_0$. We choose the arbitrary potential $V(\theta) = \cos(\theta - \gamma) + V_2(\theta)$, where γ is some phase and $V_2(\theta)$ contains only $|m| \geq 2$ harmonics. For $V_2(\theta) = 0, V(\theta)$ has symmetry center at $\theta = \gamma \mod (\pi)$, but $V_2(\theta)$ will generally make $V(\theta)$

asymmetric. However, for the given wave packets one has $C(m) = 0$ for $|m| > 1$ in (11), so that (12) and (14) reduce to expressions *completely independent* of $V_2(\theta)$:

$$\langle \hat{N} \rangle_t - \langle \hat{N} \rangle_0 = \frac{k}{2} \frac{\sin\left(\frac{\tau_\beta t}{2}\right)}{\sin\left(\frac{\tau_\beta}{2}\right)} \times \sin\left[\frac{(t+1)\tau_\beta}{2} - \Delta\gamma\right] \qquad (15)$$

and $I = -(k/2)\sin(\Delta\gamma)$, where $\Delta\gamma = \gamma - \gamma_0$. Thus, asymmetries of the potential due to $V_2(\theta)$ are totally irrelevant in determining the ratchet effect which is actually dominated by a different kind of asymmetry: the relative displacement $\Delta\gamma$ between the symmetry centers of $V(\theta)$ and $\psi_0(\theta)$. The current I vanishes only when these centers coincide ($\Delta\gamma = 0, \pi$) and assumes its largest value, $|I| = k/2$, for $\Delta\gamma = \pm\pi/2$, corresponding to "maximal asymmetry" situations.

4. Atom-Optics Experimental Realization

We now briefly describe a very recent experimental realization of the theory above [Dana *et al.*, 2008]. In this realization, a Bose–Einstein condensate (BEC) of ^{87}Rb atoms was first prepared in the initial Bloch state $\exp(i\beta x)[1 + \exp(-ix)]/\sqrt{4\pi}$, i.e. the superposition of two plane waves with momenta β and $\beta - 1$. The BEC was then exposed to a series of pulses ("kicks") from an optical standing wave, the mono-harmonic potential $V(\hat{x}) = \cos(\hat{x} - \gamma)$, where γ is an adjustable phase. With $\hat{x} \to \theta$, this kicked-particle system corresponds to the β-KR considered in the example at the end of Sec. 3: The initial Bloch state and $V(\hat{x})$ correspond, respectively, to the wave packet $\psi_0(\theta)$ with $\gamma_0 = 0$ and to the potential $V(\theta)$ with $V_2(\theta) = 0$. As in that example, we chose $g = 1$ and the kicking period was fixed to $\tau = 2\pi$ ($l_0 = q = 1$, corresponding to the "half-Talbot time"). The only resonant value of β from (4) is $\beta = 0.5$. The mean momentum $\langle \hat{p} \rangle_t$ of the BEC was measured after each kick t. It is clear from the theory above that $\langle \hat{p} \rangle_t = \langle \hat{N} \rangle_t + \beta$, implying that $\Delta\langle \hat{p} \rangle_t \equiv \langle \hat{p} \rangle_t - \langle \hat{p} \rangle_0 = \langle \hat{N} \rangle_t - \langle \hat{N} \rangle_0$. This *change* in the mean momentum (or mean velocity) is a measure of the ratchet effect induced by the kicking. One finds $\langle \hat{N} \rangle_0 = -0.5$ for the given $\psi_0(\theta)$, so that $\langle \hat{p} \rangle_0 = \langle \hat{N} \rangle_0 + \beta = 0$ and $\Delta\langle \hat{p} \rangle_t = \langle \hat{p} \rangle_t$ for $\beta = 0.5$.

The result (15), with $\Delta\gamma = \gamma$, must be adapted to the experimental conditions by taking properly into account the small but finite initial momentum width of the BEC. We assume a mixture of quasimomenta β', having a Gaussian distribution with average β and standard deviation $\Delta\beta$: $\Gamma_{\beta,\Delta\beta}(\beta') = (\Delta\beta\sqrt{2\pi})^{-1}\exp\{-(\beta' - \beta)^2/[2(\Delta\beta)^2]\}$. For small $\Delta\beta$, this is a good approximation of the actual initial momentum distribution [Davis *et al.*, 1995]. An exact formula for the average of (15) over $\beta = \beta'$ with distribution $\Gamma_{\beta,\Delta\beta}(\beta')$ can be easily derived:

$$\langle \Delta\langle \hat{p} \rangle_t \rangle_{\Delta\beta} = \frac{k}{2} \sum_{s=1}^{t} \sin(\tau_\beta s - \gamma) \exp[-2(\pi\Delta\beta s)^2]. \qquad (16)$$

Unlike (15), the expression (16) tends to a well-defined *finite* value as $t \to \infty$, for *all* β. In particular, for resonant $\beta = 0.5$,

$$\langle \Delta\langle \hat{p} \rangle_t \rangle_{\Delta\beta} = -\frac{k}{2} \sin(\gamma) \sum_{s=1}^{t} \exp[-2(\pi\Delta\beta s)^2]. \qquad (17)$$

The result (17) implies a *suppression* of the ratchet acceleration above, which is recovered as $\Delta\beta \to 0$. In practice, for sufficiently small $\Delta\beta$, the value of $\langle \Delta\langle \hat{p} \rangle_t \rangle_{\Delta\beta}$ for β close to 0.5 is much larger than that for generic β, except when $|\sin(\gamma)|$ is very small. As in the resonant case for $\Delta\beta = 0$, the ratchet effect (17) vanishes for $\gamma = 0, \pi$, when the symmetry centers of $V(\theta)$ and $\psi_0(\theta)$ coincide.

The all-optical BEC apparatus is schematically illustrated in Fig. 1. The BEC, consisting of $\sim 50\,000$ ^{87}Rb atoms, was created in a focused CO_2 laser beam and the series of optical standing-wave pulses was generated by a diode laser beam

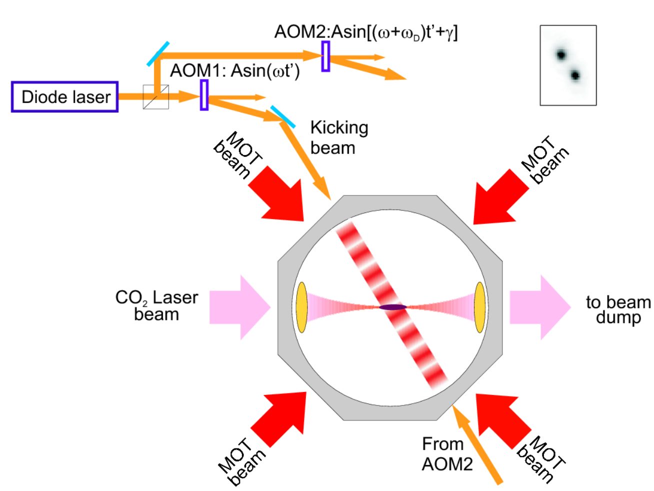

Fig. 1. Scheme of the experimental configuration.

(6.8 GHz red detuned from the 780 nm laser cooling transition) propagating at 52° to the vertical. Each of the two counterpropagating laser beams which comprised the standing wave passed through an acousto-optic modulator (AOM) driven by an arbitrary waveform generator. This enabled us to control the frequency and phase of each of the beams. Adding two counterpropagating waves differing in frequency by Δf results essentially in a standing wave that moves with a velocity v proportional to Δf; the initial momentum or quasimomentum β of the BEC relative to the standing wave is proportional to v. Thus, by varying Δf, we could set the value of β and also compensate for the effect of the gravitational acceleration along the standing wave (the experiments were done in a free-falling frame).

In order to prepare the initial state, the first standing-wave pulse was relatively long, having a duration of 38 μs. This pulse Bragg-diffracted the atoms into a superposition of two plane waves [Kozuma *et al.*, 1999]. The second and subsequent pulses of the standing wave were short enough to be in the Raman–Nath regime and enabled the realization of a kicked-rotor system. The value of the kicking strength k was measured by subjecting the BEC to one kick and comparing the populations of various diffraction orders. The value $k \sim 1.4$ was used; this corresponds, for $\tau = 2\pi$, to a *fully chaotic* classical phase space (large $K = k\tau \sim 8.8$). By varying the phase of the RF waveforms driving the AOMs, we were able to shift the position of the standing wave for the kicked rotor relative to the standing wave used in the Bragg-state preparation. This is the phase $\Delta\gamma$ or γ in Eqs. (15)–(17). Direct measurements of the initial momentum width of the BEC using a time-of-flight technique gave only an upper bound to $\Delta\beta$ of 0.1.

The experimental results are presented in Figs. 2–4 and are compared with the theory above; error bars for all the data were accurately determined by repeated measurements at fixed values of the parameters. The figures show the dependence of the mean momentum on the phase γ for $t = 5$ and resonant $\beta = 0.5$ (Fig. 2), its dependence on t for $\gamma = \pi/2$ and $\beta = 0.5$ (Fig. 3), and the dependence of the mean-momentum change on β for $t = 5$ and $\gamma = \pm\pi/2$ (Fig. 4). The solid line in the figures corresponds to either Eq. (16) or Eq. (17). The dashed line in Fig. 4 corresponds to the nonaveraged theory (15). We can see that the experimental data in all the figures is well fitted by the theory (16) or (17) for the *same* value of

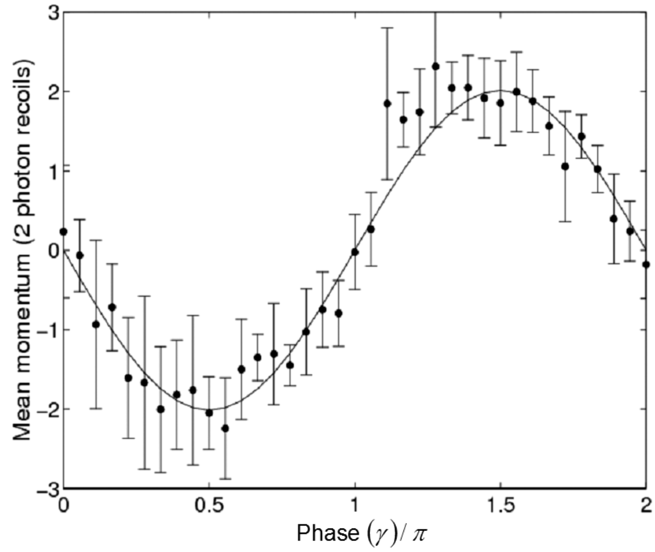

Fig. 2. Mean momentum versus phase angle γ for kicking strength $k = 1.4$, kick number $t = 5$, and resonant quasimomentum $\beta = 0.5$. The filled circles and associated error bars are from the experiment. The solid line corresponds to the theory (17) for a BEC with width $\Delta\beta = 0.056$. Notice how the current direction can be easily reversed by varying γ.

the width $\Delta\beta$, $\Delta\beta = 0.056$. This value of $\Delta\beta$ is also consistent with what can be measured directly using time-of-flight. These facts indicate good agreement of the experimental results with the QR-ratchet theory above. Thus, the clear saturation of the mean momentum in Fig. 3 provides experimental evidence for the suppression of the resonant ratchet acceleration. The inset in Fig. 3 plots the time-of-flight images of the kicked BEC as time increases. Note

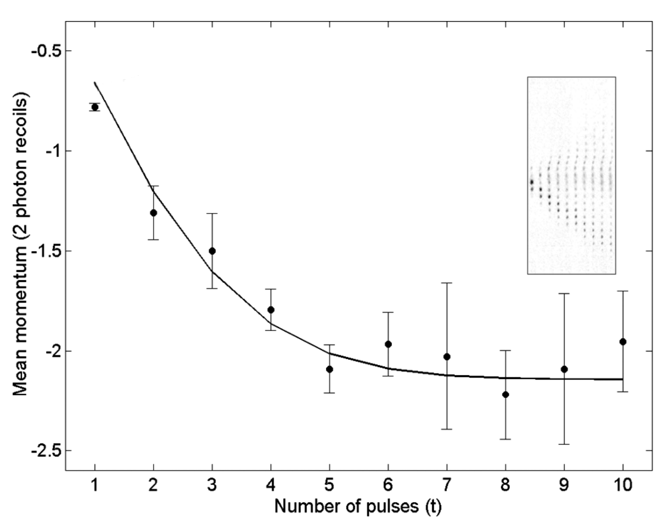

Fig. 3. Mean momentum versus kick number t for $k = 1.4$, $\gamma = \pi/2$, and $\beta = 0.5$. The filled circles and error bars are experimental data and the solid line corresponds to Eq. (17) ($\Delta\beta = 0.056$).

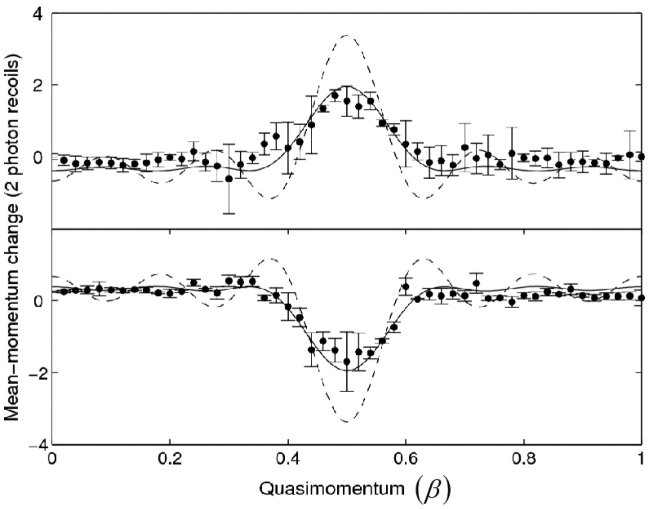

Fig. 4. Mean-momentum change after $t = 5$ kicks versus β for $k = 1.4$. In the upper (lower) plots, $\gamma = -\pi/2$ ($\gamma = \pi/2$). The filled circles and error bars are experimental data; the dashed and solid lines correspond, respectively, to Eqs. (15) and (16) ($\Delta\beta = 0.056$).

that the momentum distribution is not symmetric and is weighted towards the negative diffraction orders, as expected from the mean-momentum values. Figure 4 shows that the saturated value of the mean momentum (the ratchet velocity) around resonant $\beta \sim 0.5$ is still strongly pronounced relative to that for nonresonant β.

5. Conclusions

In conclusion, we have presented a theory of QR ratchets for the periodically kicked particle, focusing on the main QRs. We have then described a very recent atom-optics experimental realization of a special case of this theory [Dana *et al.*, 2008]. This case corresponds to primitive QRs with $g = 1$ and to simple kicking potential $V(\theta)$ and initial wave packet $\psi_0(\theta)$, both having a point symmetry. Despite this symmetry, the QR-ratchet effect emerges from the relative asymmetry associated with the displacement $\Delta\gamma$ between the symmetry centers of $V(\theta)$ and $\psi_0(\theta)$. We emphasize that the quantities $I = -(k/2)\sin(\Delta\gamma)$ and (15)–(17), depending on the phase $\Delta\gamma$ (or γ), are of a *purely quantum* nature due to the relatively large value of the effective Planck constant, $\tau/(2\pi) = 1$. These quantities are therefore basically different from trivial currents arising from biased phases in simple classical systems (see e.g. [Goychuk & Hänggi, 2001]).

In fact, for $g = 1$ one can write (14) as $I = \int_0^{2\pi} f(\theta)|\psi_0(\theta)|^2 d\theta$, where $f(\theta) = -kdV/d\theta$ is the force (see also [Lundh & Wallin, 2005]). If one interprets $|\psi_0(\theta)|^2$ as the distribution function in θ of an initial classical ensemble of particles having all the same momentum, then I measures the "initial-kick" effect, i.e. the mean momentum change of the ensemble after one kick. After not many kicks, however, the mean momentum generally saturates to a finite value [Dana *et al.*, 2008]. On the other hand, the quantum mean momentum increases linearly in time with rate I, as if the "initial-kick" effect *persists coherently for all times.* This is thus a genuine quantum-ratchet acceleration having *no* classical explanation. Also, the saturation of the quantum mean momentum due to the finite momentum width of the BEC [Eqs. (16) and (17)] has absolutely *no* relation with the classical saturation above [Dana *et al.*, 2008]. All this clearly shows the purely quantum nature of the QR-ratchet effect already in the simple case of $g = 1$. For $g > 1$ and/or general QR ratchets, even the classical "initial-kick" interpretation of I is *not* possible. An extreme case is that of a (usually asymmetric) potential having only harmonics of order $|m| < g$; then, (14) implies that $I = 0$ (quantum antiresonance), i.e. *no* "initial-kick" effect for *any* initial wave packet.

We plan an experimental realization of QR ratchets for the periodically kicked particle under gravity (or arbitrary linear potential) in the free-falling frame (FFF). This system has attracted much interest recently since it exhibits the experimentally discovered "quantum accelerator modes" (QAMs) in the vicinity of main QRs [Oberthaler *et al.*, 1999]. Because gravity is not "felt" in the FFF, the particle is again subjected to an unbiased force and one can thus speak about ratchets. However, the noninertial nature of the FFF leads to QR ratchets with interesting properties having no analogues in the nongravity case, as shown in a recent theoretical study [Dana & Roitberg, 2007]. In particular, the QR-ratchet characteristics were found to depend significantly on number-theoretical features of the gravity parameter. While QAMs (*near* main QRs) are associated with accelerator-mode islands [Fishman *et al.*, 2003], main QR-ratchet effects (*at* QR) are essentially independent on the nonintegrability parameter and therefore occur also in fully chaotic regimes. The transition between these two different kinds of quantum directed transport as parameters such as k and τ are properly varied is a fascinating and important problem, from both the theoretical and experimental viewpoints.

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