**Bar-Ilan Meeting on**
**ADVANCES IN CLASSICAL AND QUANTUM CHAOS**

# ABSTRACTS

## Pseudochaotic Dynamics: Old, New, and Awful

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We define pseudochaos as a random dynamics with zero Lyapunov exponent. Randomness is considered as a sensitive motion that can be described using some principles of statistical ensembles, kinetics, and distributions. Many examples of chaos in realistic systems have features of pseudochaos. The most important property of pseudochaos is persistent fluctuations, which makes impossible to apply usual statistical description and requires new approaches of the fractional kinetic type. Different examples will be provided to show this kind of randomness that can be considered as intermediate between regular dynamics and chaos.

## Steep Billiard Potentials

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The behavior of a point particle traveling with a constant speed in a region $D$, undergoing elastic collisions at the region's boundary, is known as the billiard problem. In many applications (e.g., molecular dynamics, cold-atoms optical traps, microwaves in the semi-classical limit), the billiard's flow is a simplified model which imitates the conservative motion of a particle in a smooth steep potential $V_\epsilon$, which, in the small $\epsilon$ limit, becomes a hard-wall potential. Indeed, one of the underlying assumptions of Boltzmann ergodic hypothesis is that molecules behave like hard spheres.

We study rigorously this steep potential limit for arbitrary geometry and dimension; on one hand, for regular reflections, under some natural assumptions on the potentials, we provide the asymptotic expansion of the smooth solutions in terms of auxiliary billiard approximations, with error estimates which are small and have small derivatives. On the other hand, in two dimensions, we prove that tangent periodic orbits and corner polygons produce stability islands in arbitrary geometry, even in the dispersing case for which the billiards are mixing. Partial generalizations to the $n$-dimensional case are emerging: we proved recently that linearly stable periodic orbits appear in arbitrarily steep smooth power-law potentials which limit to specific $n$-dimensional dispersing Sinai billiards. Thus, smooth dispersing arbitrarily steep potentials may produce non-ergodic flows in arbitrary large dimension.

Joint work with A. Rapoport and D. Turaev.

**Classical versus Quantum Mechanics in Two-Electron Atoms - from Quantum Spectra to Photo-Ionisation Cross Sections**

Gregor Tanner
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Gregory H Wannier suggested in 1953 that the energy dependence of the double-ionisation cross section in two-electron atoms is close to the threshold related to the classical dynamics near the triple collision singularity. It took 35 years to confirm these ideas experimentally and they represent one of the finest examples of how quantum correlations can be described in terms of classical phase space arguments.

In this talk I will give an overview over some more recent results using semiclassical techniques in this three-particle Coulomb problem. I will focus on the energy regime below the double-ionisation threshold where the classical dynamics becomes chaotic and the quantum spectrum is formed by complicated resonance patterns. The key to a better understanding of the high-dimensional phase space dynamics are the stable and unstable manifolds of the non-regularisable triple collision which can be analyzed using McGehee scaling techniques developed in the context of celestial mechanics. We use this information to extend Wannier's ideas to the energy regime below the three-particle breakup threshold $E < 0$ describing the asymptotics $E \to 0_-$. Access to information in this semiclassical limit is hard to obtain due to the predominantly chaotic nature of the classical dynamics.

I will show that universal scaling laws with exponents described in terms of so-called Siegel-exponents of the triple collision dominate the amplitude of the fluctuations in the photo-ionisation cross section for single ionisation in the limit $E \to 0_-$. This exponent is different from Wannier's exponent valid for $E \to 0_+$. Our approach leads naturally to a generalization of semiclassical closed orbit theory to non-regularisable singularities at the origin. In particular, the fluctuations in the cross section can be linked to a set of infinitely unstable classical orbits starting and ending in the triple collision. The findings are compared with quantum calculations for a model system, namely collinear helium.

**Quantum Computation and Quantum Chaos**

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Quantum computation in presence of realistic imperfections is considered. The effects of imperfections on the accuracy of various quantum algorithms are analyzed and links with quantum chaos and random matrix theory are established. Efficiency of quantum algorithms for complex dynamics is investigated. Recent experimental implementations of proposed quantum algorithms are also discussed.

**Tunneling Out of Phase-Space Islands of Maps**

Shmuel Fishman
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Experimentally observable Quantum Accelerator Modes are used as a test case for the study of some general aspects of quantum decay from classical stable islands immersed in a chaotic sea. Different regimes of tunneling, marked by different quantitative dependence of the lifetimes on the effective Planck's constant, are identified, depending on the resolution of KAM substructures that is achieved on its scale. The theory of Resonance Assisted Tunneling is revisited, and found to well describe decay whenever applicable. Relevance for various physical systems will be discussed.

**Random Discrete Schrödinger Operators from Random Matrix Theory**

Uzy Smilansky
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We investigate random, discrete Schrödinger operators which arise naturally in the theory of random matrices, and depend parametrically on Dyson's Coulomb gas inverse temperature $\beta$. They belong to the class of "critical" random Schrödinger operators with random potentials which diminish as $|x|^{-1/2}$. We show that as a function of $\beta$ they undergo a transition from a regime of (power-law) localized eigenstates with a pure point spectrum for $\beta < 2$ to a regime of extended states with irregular continuous spectrum for $\beta \geq 2$.

**Diffractive Energy Spreading and its Semiclassical Limit**

Doron Cohen
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We consider driven systems where the driving induces jumps in energy space: (1) particles pulsed by a step potential; (2) particles in a box with a moving wall; (3) particles in a ring driven by an electro-motive-force. In all these cases the route towards quantum-classical correspondence is highly non-trivial. Some insight is gained by observing that the dynamics in energy space, where $n$ is the level index, is essentially the same as that of Bloch electrons in a tight binding model, where $n$ is the site index. The mean level spacing is like a constant electric field and the driving induces long range hopping $1/(n-m)$.

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